Excitation and propagation of plasma waves excited by runaway electrons

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12th West Lake International Symposium on Plasma Simulation
May 3 2018

Highlights

- A quasilinear simulation model is developed to study the wave-particle interaction of runaway electrons and plasma waves in tokamaks.
  - Due to bump-on-tail and anisotropicity of runaway electron distribution in momentum space, whistler waves and extraordinary electron waves can be driven unstable.
  - Excited waves cause diffusions through resonances, and strongly alter runaway electron distribution in momentum space.
- Propagation of excited waves is studied using ray-tracing method.
  - Because of density cutoff, only the very low frequency whistler waves ($<$ 200MHz) and extraordinary waves with $\omega \approx \omega_{pe}$ can propagate to the edge and get detected.

Outline

1. Motivation: Direct observations of plasma waves excited by runaway electrons
2. Simulation model
3. Simulation results
4. Role of excited waves on RE avalanche and electron cyclotron emission
5. Propagation and oscillatory behavior of excited waves
6. Summary

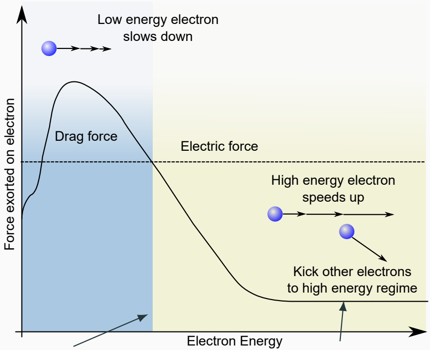
Motivation: Direct observations of plasma waves excited by runaway electrons

Basic pictures of runaway electrons (REs)

- In plasma, drag force on electrons due to Coulomb collision is a non-monotonic function of p. For $v \gg v_{th}$, collision frequency$\sim v^{-3}$, collisional drag force decrease with p.

- With $E > E_{CH}$ (Connor-Hastie field), electrons with momentum larger than p_{crit} can run away to higher energy.
- Knock-on collision of high energy electron with thermal electron can lead to avalanche growth of RE.

- Runaway electrons can be generated in tokamak experiments in both flattop phase and post-disruption scenarios.



runaway-loss separatrix

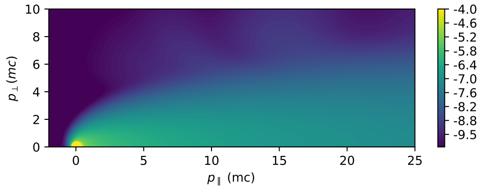
Connor-Hastie critical E

- Large population of energetic REs (10$\sim$ 100 MeV) can be generated in disruptions, which can cause damage to tokamak device.

Runaway electrons are susceptible to various kinds of kinetic instabilities

- REs can excite kinetic instabilities, including whistler waves (WW) and extraordinary electron waves (EXEL), through wave-particle resonances.

- Bump-on-tail $\rightarrow$ Cherenkov resonance $(\omega - k_{\|} v_{\|} = 0)$
- Anisotropy $\rightarrow$ Doppler resonance. $(\omega - k_{\|} v_{\|} = n\omega_{ce}/\gamma)$
- The excited modes can cause energy diffusion and pitch angle scattering of resonant electrons.



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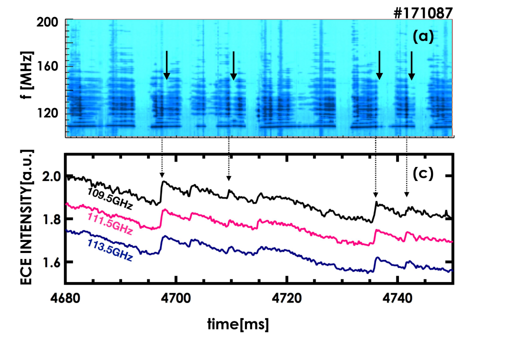
T. Fülöp, G. Pokol, P. Helander, and M. Lisak, Physics of Plasmas 13, 062506 (2006).
G.I. Pokol, A. Kómár, A. Budai, A. Stahl, and T. Fülöp, Phys. Plasmas 21, 102503 (2014).
P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

Whistler (helicon) waves are observed in recent RE experiments on DIII-D

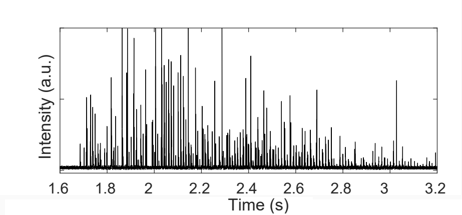
- In DIII-D flattop RE experiments, whistler with frequency range 100-200 MHz ($\omega_{ci} < \omega < \omega_{LH}$) are observed directly at plasma edge.
- Wave amplitudes show bursting behavior, which is correlated with the ECE signals.
- Discrete structure is found in the wave spectrum.
- Similar phenomena are observed in disruption experiments in DIII-D.

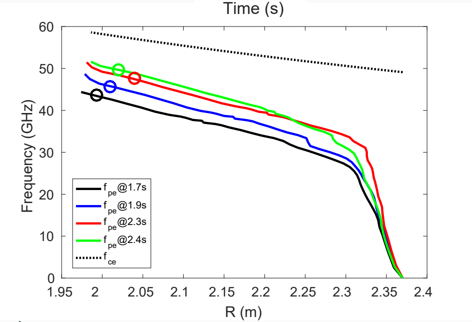


D.A. Spong et al., Phys. Rev. Lett. 120, 155002 (2018)

Intermittent RF radiation at core plasma frequency are observed in EAST tokamaks

- In recent runaway electron experiments in EAST, bursts of RF radiation with frequency close to the core plasma frequency (ω_{pe}) are observed near the edge.
- Similar phenomena have been observed in C-Mod experiments.
- Amplitudes of the waves are significantly higher than thermal radiation like cyclotron emission ($T_{\mathrm{rad}} \sim 10^5$ keV)





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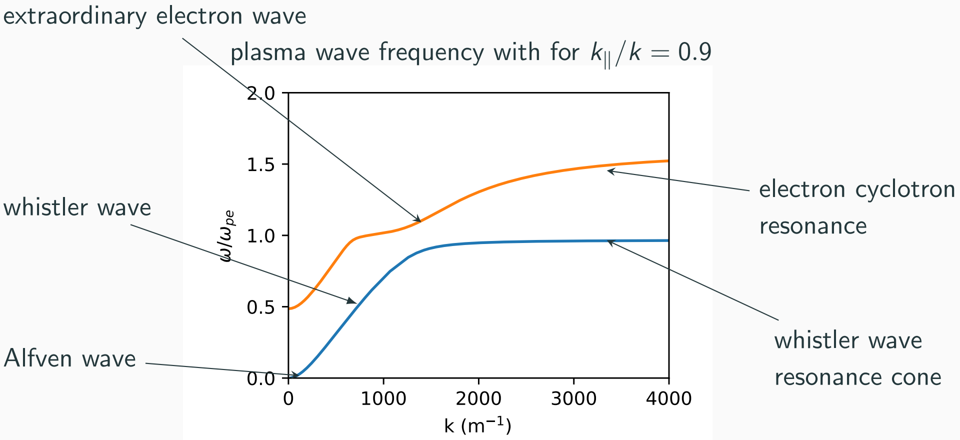
R.F. Gandy, I.H. Hutchinson, and D.H. Yates, Phys. Rev. Lett. 54, 800 (1985).

Simulation model

Both whistler waves and extraordinary electron waves are included in the simulation model

- Wave frequencies are calculated using cold plasma dispersion relation, including both electron and ion cyclotron resonances and whistler wave resonance.



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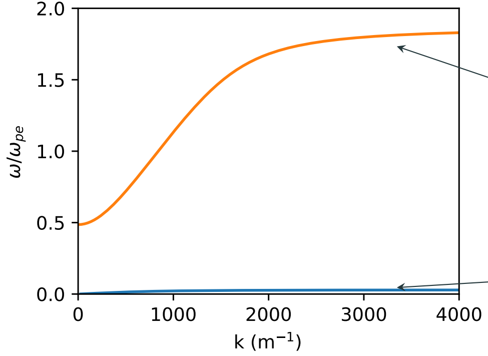
plasma wave frequency with for $k_{||}/k = 0.1$



upper hybrid resonance

lower hybrid resonance

RE distribution in momentum space is calculated dynamically

RE distribution evolution is calculated by solving the kinetic equation with time, in 2D momentum space (after gyro-averaging) using finite-element method.

collision operator

$$\frac{\partial f}{\partial t} + \frac{eE_{\|}}{mc}\left(\xi \frac{\partial f}{\partial p} + \frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi}\right) + C\left[f\right] + \frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathbf{F}_{\mathrm{rad}} f\right) + D\left[f\right] = S_A\left[f\right]$$

electric field force

synchrotron radiation reaction force

wave diffusion

RE avalanche source term

Mode amplitudes are calculated according to linear growth rate and background thermal fluctuations

linear growth rate

$$\frac{d\mathcal{E}(k,\omega)}{dt} = 2\Gamma(k,\omega)\mathcal{E}(k,\omega) + \mathcal{K}(k,\omega)$$

wave energy

thermal fluctuation

Growth of unstable modes are calculated according to RE distribution

$$\Gamma = \frac{\omega_{pe}^2}{\mathcal{D}} \int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi \delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma) \left[v\frac{\partial f}{\partial p} - \frac{v}{p} \frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi} \frac{\partial f}{\partial \xi} \right]$$

$$Q_n = \left[E_x \frac{n\omega_{ce}}{\gamma k_\perp v} J_n(k_\perp \rho) + E_z \xi J_n(k_\perp \rho) + iE_y\sqrt{1-\xi^2} J_n'(k_\perp \rho) \right]^2$$

$$\mathcal{D} = \frac{1}{\omega} \mathbf{E}^* \cdot \frac{\partial}{\partial \omega}(\omega^2 \epsilon) \cdot \mathbf{E} \qquad \xi \text{ is the cosine of pitch angle}$$

- For $n = 0$ (Cherenkov): Γ depends on $\partial f/\partial p_\parallel$, Landau damping & bump on tail
- For $n < 0$ (Normal Doppler): Anisotropic distribution ($\partial f/\partial \xi > 0$) stabilize the mode
- For $n > 0$ (Anomalous Doppler): Anisotropic distribution gives positive growth rate

The growth rate Γ is subtracted by the damping rate due to collisions.

P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

Diffusion of RE in momentum space is addressed using quasilinear model

$$D[f] = \frac{1}{2}e^2 \sum_{n=-\infty}^{\infty} \int d^3\mathbf{k}\, \hat{L}[p_\perp \delta(\omega - k_{||} v\xi - n\omega_{ce}/\gamma)|\psi(n, \mathbf{k}, \omega)|^2 p_\perp \hat{L} f]$$

$$\hat{L} f = \frac{1}{p}\frac{\partial f}{\partial p} - \frac{1}{p^2}\frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi}\frac{\partial f}{\partial \xi}$$

$$\psi(n, \mathbf{k}, \omega) = \frac{1}{2}(E_x + iE_y)J_{n-1}(k_\perp \rho) + \frac{1}{2}(E_x - iE_y)J_{n+1}(k_\perp \rho) + \frac{p_{||}}{p_\perp}E_z J_n(k_\perp \rho)$$

We only take into account $n = 0$ and $n = \pm 1$, which are the dominant resonances of whistler waves.

A. N. Kaufman, "Quasilinear diffusion of an axisymmetric toroidal plasma," Phys. Fluids 15, 1063 (1972).

Propagation of the excited waves are calculated using geometrical optics (ray-tracing)

Geometrical optics approximation is valid for most of the whistler waves and extraordinary electron waves, which satisfies $\lambda \ll L$.

$$\frac{dk_r}{dt} = -\frac{\partial\omega}{\partial r} \qquad \frac{\partial k_z}{\partial t} = -\frac{\partial\omega}{\partial z}$$

$$\frac{dr}{dt} = \frac{\partial\omega}{\partial k_r} \qquad \frac{dz}{dt} = \frac{\partial\omega}{\partial k_z}$$

$$rk_\phi = \mathrm{const}$$

For very low frequency whistler waves (< 200 MHz), λ is comparable to minor radius and the ray-tracing method may be not sufficient. A full-wave treatment is required.

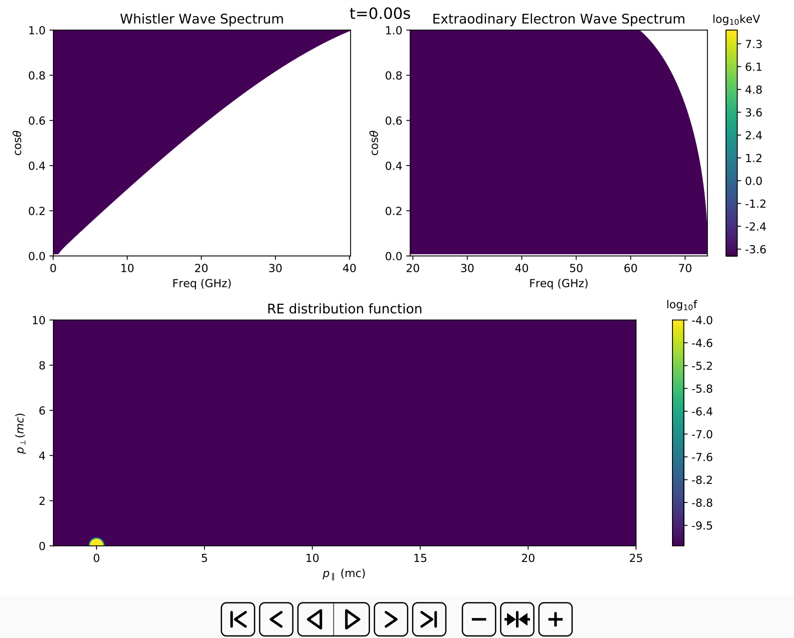
GENRAY, A. Smirnov, R. Harvey, and K. Kupfer, Bull Amer. Phys. Soc., Vol. 39 p. 1626.

Simulation results

RE growth in flattop phase of discharge with low electron density

$$n_e = 2.0 \times 10^{19}\mathrm{m}^{-3} \quad B = 2T \quad T_e = 1.1\mathrm{keV} \quad E/E_{CH} = 9$$

- With low electron density and $E > E_{CH}$ (Connor-Hastie), the runaway electron population can grow through both Dreicer generation (slide-away of high-energy electrons from Maxwellian tail) and the avalanche (knock-on collisions).
- Whistler modes get excited after RE population reached certain threshold.
    - This threshold is mostly determined by the Landau damping rate.


t=0.00s
Whistler Wave Spectrum
Extraodinary Electron Wave Spectrum
cosθ
Freq (GHz)
log10keV
7.3
6.1
4.8
3.6
2.4
1.2
0.0
-1.2
-2.4
-3.6
RE distribution function
log10f
-4.0
-4.6
-5.2
-5.8
-6.4
-7.0
-7.6
-8.2
-8.8
-9.5
p⊥ (mc)
p∥ (mc)

Both the whistler waves and the extraordinary electrons waves are excited during simulation

For whistler waves,

- Low frequency whistler waves (LFWW, 1GHz-10GHz) first get excited, and scatter RE in high energy regime ($15 < \gamma < 20$, fan instability).
  - Stop RE from going into higher energy regime
- High frequency whistler waves (HFWW, 10GHz-40GHz, close to resonance cone) get excited later, and scatter RE in low energy regime ($2 < \gamma < 5$).
- Very low frequency whistler waves ($<$ 200 MHz) also get excited in the later time.

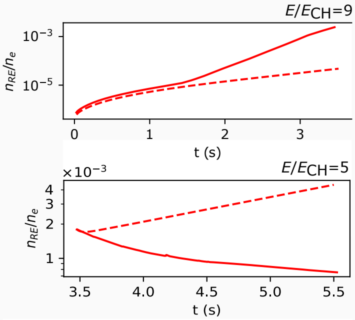
For extraordinary electron waves,

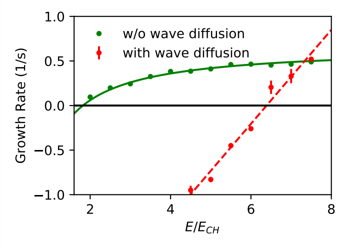
- When whistle waves are excited, RF waves with frequency close to plasma frequency and $k_{||} \gg k_{\perp}$ also get excited.

Role of excited waves on RE avalanche and electron cyclotron emission

Role of excited waves on RE avalanche and critical electric field

- The excited waves can cause both energy diffusion and pitch angle scattering of REs.
- For $E \gg E_{CH}$, energy diffusion enhances the RE avalanche growth.
    - Resonance region of excited waves overlaps with the runaway-loss separatrix, so energy diffusion causes more electrons entering the runaway region.
- For smaller E field, overlapping does not happen and pitch-angle scattering suppresses the avalanche growth.
- Competition of two effects results in a higher critical electric field than predicted by Rosenbluth-Putvinski theory, which is much closer to experimental observations.





C. Liu, E. Hirvijoki, G. Fu, D.P. Brennan, A. Bhattacharjee, and C. Paz-Soldan, ArXiv:1801.01827 (2018).

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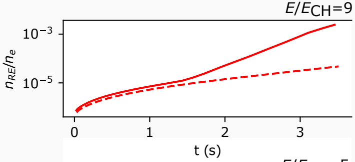
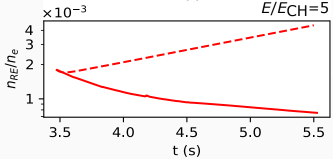
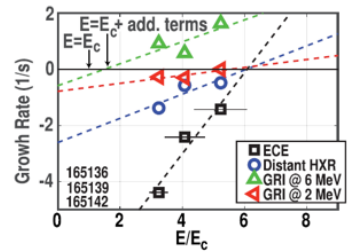






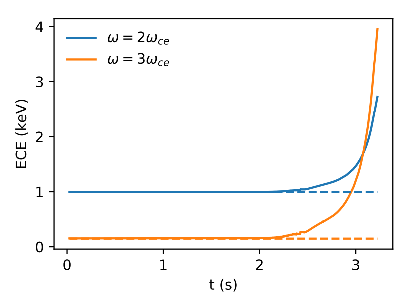
C. Liu, E. Hirvijoki, G. Fu, D.P. Brennan, A. Bhattacharjee, and C. Paz-Soldan, ArXiv:1801.01827 (2018).

Effects of excited waves on the electron-cyclotron-emission (ECE) signals

- The pitch-angle scattering transfer electrons' momentum from $p_{||}$ to $p_{\perp}$, which can enhance the power of electron-cyclotron emission.
- We develop a new ECE synthetic diagnostic tool for runaway electrons, in order to benchmark with experiments.

Results:

- ECE signals from REs start to grow abruptly after the high frequency whistler waves are excited, and can overwhelm the ECE from thermal electrons.
- Signals at higher frequencies is more enhanced than lower frequencies.



C. Liu, L. Shi, E. Hirvijoki, D.P. Brennan, A. Bhattacharjee, C. Paz-Soldan, and M.E. Austin, ArXiv:1803.09897 (2018).

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Results:

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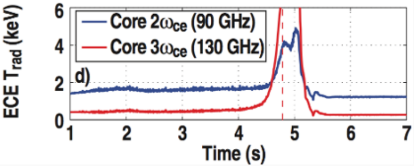
- Signals at higher frequencies is more enhanced than lower frequencies.
- The results match well with DIII-D ECE diagnostic results.



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Propagation and oscillatory behavior of excited waves

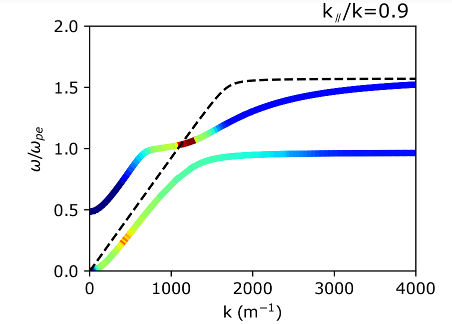
Can excited waves propagate to the edge?

The excited waves by RE have wide frequency range, but why only the very low frequency whistler waves (<200MHz) and RF waves with $\omega \approx \omega_{pe}$ are observed?

- Some of the waves generated at core (where most of REs lie) cannot propagate to the edge due to density cutoff.
- Existing diagnostic methods (Langmuir probes, RF antenna, spectrometer) cannot measure the waves in certain frequency range.

The density cutoff can be estimated using $rk_\phi = const$ and $k_\perp = 0$.

- Dashed line represents cutoff dispersion, and only waves above dashed line can propagate to the edge.
- Color represents amplitudes of excited waves



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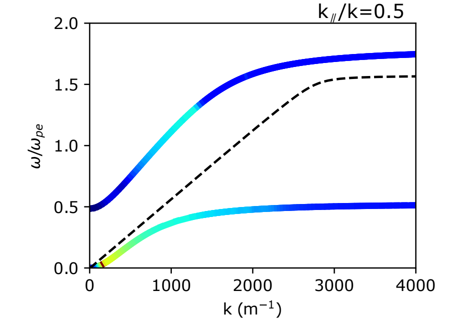

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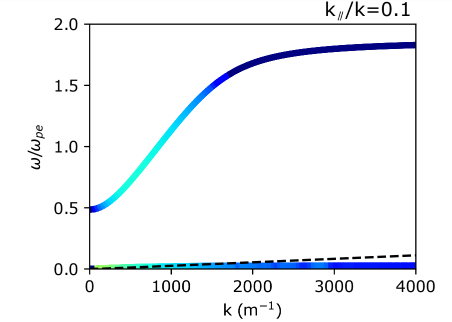

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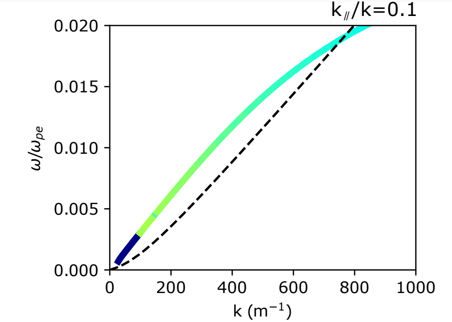
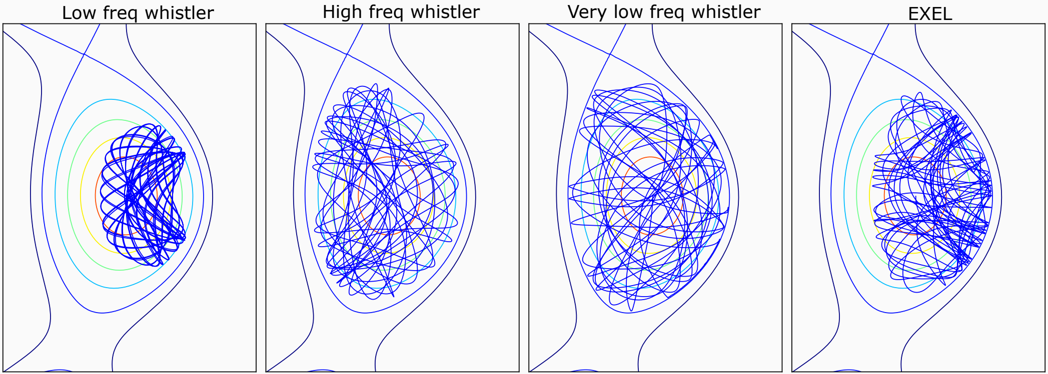


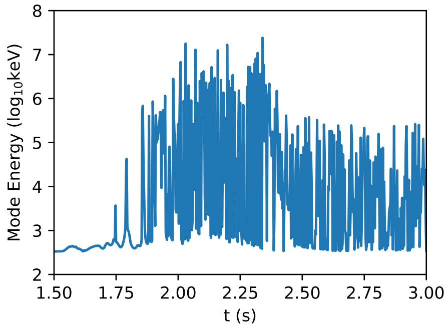
Ray-tracing calculations of excited waves



- The propagation of both the low frequency whistle waves and high frequency whistler waves are limited by density cutoff.
- The very low frequency whistler waves (<200MHz) and the EXEL waves can propagate to the edge on the high-field-side.

Oscillatory behavior of excited EXEL waves

- Time evolution of EXEL wave amplitude from simulation shows oscillatory behavior.
  - Related to the excitation-saturation nonlinear cycle and the interaction between the plasma waves and the RE distribution (predator-prey model).
  - Can explain intermittent bursts of RF waves observed in EAST experiments.
- Similar oscillatory phenomena of very low frequency whistler waves are also observed in DIII-D experiments.



Summary

Summary

- Using a self-consistent quasilinear simulation model, we find that the RE tail generated in tokamak experiments can excite whistler waves in both low frequency and high frequency range, and extraordinary electron waves with $\omega \approx \omega_{pe}$.
- Excited waves can significantly affect the RE avalanche and the electron cyclotron radiation.
- Due to density cutoff, only the very low frequency whistler waves and EXEL waves can propagate to the edge.
- Wave amplitude shows oscillatory behavior.
- Future work:
  - Extend distribution function calculation from 2D to 3D using bounce-average model.

Thank you

Backup slides

Backup